Finger Counting as a Calendar: a case of distributed cognition

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Abstract

According to the distributed cognition view[2], human cognitive processes are interactive processes between internal and external representations. External representations and their manipulations are part of cognitive processes. In order to facilitate cognitive processes, external representations should be coordinated well with human visual and motor systems. Investigation of external representations provides the insight into human cognitive processes. This paper aims to discuss the relationship between external representation and the visual and motor systems by introducing an illustrative example, hand-held calendar from Japan which we call "finger counting".

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1 Introduction

The objective of this paper is to discuss the relationship between external representation and the visual and motor systems. We introduce an interesting example from lore in Japan, which we refer to as "finger counting"[1]. "Finger counting" computes the day of the week from a given date without reference to a regular calendar. This paper will provide a computational account of "finger counting". We will show how "finger counting" achieves the same computation as a calendar[2] in a different way and how "finger counting" utilizes the characteristics of the motor system rather than the visual system. "Finger counting" illustrates the nature of cognitive processes in terms of the interaction between external and internal representations.

If someone were to ask us what day is 24th December, we would probably use a calendar. If someone were to ask us to calculate 1234×5678, we would use a pen and a piece of paper, or possibly a calculator. Looking at a calendar, manipulating numbers with a paper and a pen, or punching the numbers in calculator yields answers. From the distributed cognition view[2], human cognitive processes are the interactive processes between internal and external representations. Traditional views of cognition focus on manipulations of internal representations alone "in the head", whereas we argue for another view. Manipulations of external and internal representations are interwoven with each other in the process so that the cognitive process changes as external representations change.[6]. Parts of cognitive processes are substituted or facilitated by altering external representations[3], [4].

To facilitate cognitive processes generally, external representations should be coordinated well with other cognitive subprocesses such as visual and motor systems[3]. Thus,

[1]At the present time, we don't know much about how "finger counting" has been historically constructed. We know, I Chang, a Chinese future-reading, has similar uses of fingers. Also, "finger counting" is called as "the wisdom of grandma" in several parts of Japan. It is interesting to investigate its historical construction but beyond a scope of this paper.

[2]In this paper, I use "calendar" to refer to a Western calendar system, or the Gregorian calendar, which, it is important to realize, is the product of many generations of cultural refinement.

[3]It is interesting to observe that external representations, even those which we tend to think of as so intrinsically natural and given, often have a long history to obtain modern forms (e.g. numbers of the Arabic system[5], ship navigation [2]).

investigation of characteristics of external representations and of the way they are used contributes to the understanding of cognitive subprocesses and their interactions, that is, cognitive processes.

2 Western Calendar System

In this section, we introduce the characteristics of the Western calendar system which we will contrast later with the "finger counting" method. We will refer to "day" as to mean days of the week and to "date" as to mean a calendar date.

Calendars are external representations that coordinate well with properties of human visual system. A calendar represents the relationship between the day and the date of *each month* of *each year* so that a human does not have to build a complex mapping between a day and a date of a month of a year. Conventional calendars form a matrix with columns of 7 days of the week. The convention that Sunday is in the left most column makes it easier to map the position of a date in the matrix to a day of the week regardless of the month. The matrix form facilitates visual search in two ways: (1) it maps a date to a position in the matrix for the month and (2) it maps the date to a day of the week using the position. The matrix form of calendars and the convention is coordinated well with properties of the visual system such as visual search.

2.1 The "Day" System

A week is composed of 7 days *with no exception.* The set of 7 days is specificly pre-ordered, needless to say: Sunday, Monday, Tuesday, etc.. This regularity is not interfered with by any other aspects of the date system, meaning that the number of days does not change from week to week.

2.2 The "Date" System

A year is usually composed of 365 days and each day is classified into one of 12 months.[4] There is a regularity in the pattern across months but it is not as straight forward as the pattern in the day system. The number of days in the month does not simply alternate between 30 and 31 days and every 4 years a day is added to the month of February. Such an irregularity changes the relationship between days of the week and calendar dates month after month and year after year.

2.3 The Relationship between Date and Day Systems

Any date of *any month* of *any year* belongs to one of 7 days of the week. To use this regularity in the computation of getting a day from a date, we need two pieces of information: (1) *the classification of a date to one of 7 abstract classes*[5] *by modulo 7,* and (2) *the alignment between these classes and the days of the week*[6]. For example, suppose April 1st of 1997 falls on a Tuesday, and you want to know what day of the week will August 18th of 1997 be[7]. One way to compute this is as follows:

> *April has 30 days, May 31 days, June 30 days, July 31 days, and 18 days of August. So,* $30+31+30+31+18 = 140$*. Now, we have 140 days counting from April 1st. Since each week has 7 days,* $140 \div 7 = 20$*, that is,* $0 = 140\,(mod\,7)$*. Since April 1st is a Tuesday, that is, Tuesday* $= 1\,(mod\,7)$*, then, Monday* $= 0\,(mod\,7)$*. Therefore, August 18th falls on a Monday as well.*

In the calculation above, we actually convert the irregularity of date systems, or different numbers of days of months, into the regularity of day system by counting the number

[4]Some months consist of 30 days, others of 31 days, and February usually has 28 days except in a leap year when it has 29 days. If we count the number of days for each month, it is $(31, 28(29), 31, 30, 31, 30, 31, 31, 30, 31, 30, 31)$.

[5]In mathematics, it is said that any natural numbers belong to one of congruent classes of modulo 7. It can be written as $a = b\,(mod\,7)$, where $a \in \{0, 1, 2, 3, 4, 5, 6\}$ and b is any natural number. Or we can say a and b leave the same remainder when divided by 7. To avoid using mathematical terms, here, we will refer to "one of congruent classes of modulo 7" as "one of (abstract) classes by modulo 7".

[6]According to the footnote 5, classifiers are $\{0, 1, 2, 3, 4, 5, 6\}$.

[7]We will use August 18th, 1997 is an example through this paper. I wanted to pick a date of "1996" as an example but unfortunately "1996" is a leap year. In order to avoid the potential confusion, we adopt "1997" instead of "1996".

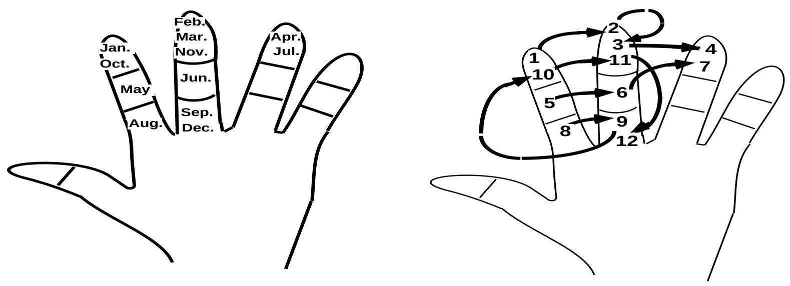

Figure 1: "Finger Counting" Structure of Month: (left) the mapping of months onto finger sections (right) the sequential structure of months

of days from April 1 and dividing this number by 7 (i.e. the information (1) above). Then, using the information of the mapping of the specific date with the day of the week, we aligned abstract classes by modulo 7 with 7 days of the week (i.e. the information (2) above). Our calculation relies on the regularity of the day system.

3 Finger Counting

In this section, we explain structures and representations of "finger counting" and also its procedure.

3.1 Structures and Representations

3.1.1 The Month and Date Structure

See Figure 1. Figure 1 (left) shows the mapping of each of the 12 months onto a section of the fingers. We use only 7 sections. three of the first finger, three of the second finger, and one of the third finger. Figure 1 (right) shows the sequence of 12 months in order. This mapping between months and sections is constant across years.

3.1.2 The Day Structure

Each year, the mapping of days onto finger sections is going to be fixed.[8]. See Figure 2. Figure 2 (left) shows the mapping of 1997. In the beginning of each year, we map the

[8]In a leap year, we need a small modification.

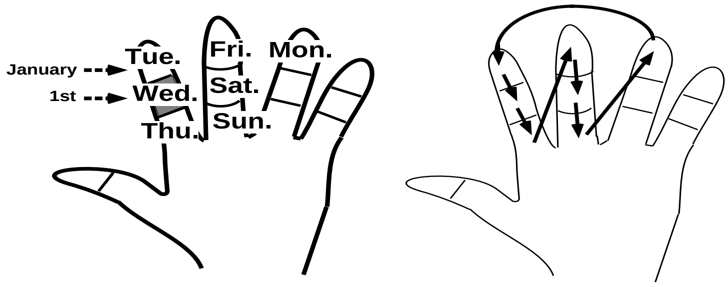

Figure 2: "Finger Counting" Structure of Day: (left) the mapping of 7 days of 1997 onto finger sections (right) the sequential order in the alignment of the 7 days with the 7 sections

day of January 1st onto the shaded area. Once we fix this mapping, the alignment of other days and other sections are easily determined as in the sequential order of Figure 2 (right).

3.2 The Procedure

Now, let us explain the procedure of this computation. First, we go through an example and then we give the general account of the procedure.

3.2.1 An Example: what day is August 18, 1997?

Suppose we want to know the day of August 18th, 1997. The procedure is as follows (also see the legend of Figure 3):

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1. We go to the section of August as shown in Figure 1.
2. We get the class of the date, 18, by modulo 7, that is 18 = 4 mod 7
3. From the section of August, which we count as 0th place of August,
   we go to the 4th place of August as in Figure 3.
4. The 4th place, the place of August 18, is Monday according to the
   mapping between days and sections as shown in Figure 2.
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3.2.2 The General Procedure

The general procedure of "finger counting" is as follows:

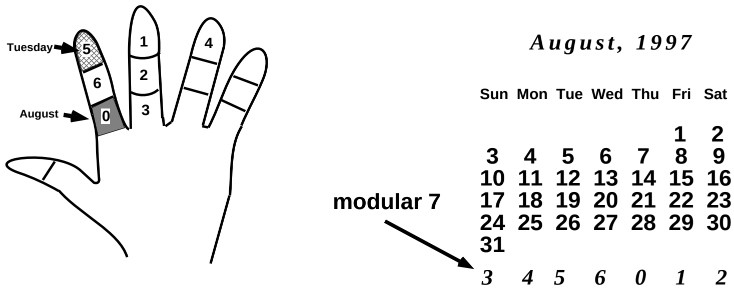

Figure 3: Example of the Calculation of August 18th, 1997: (left) by the mapping of months to finger sections shown in Figure 1, the section of August can be obtained. Numbers of sections in this figure is classes by modulo 7 *counted from the section of August*. As an example, 18th of August is 4 by modulo 7, that is, $4 = 18\,(\,mod\,7)$, so that August 18th is allocated as the class '4' in this figure. Given the knowledge of Tuesday of 1997 allocated at the tip section of the first finger, we can know August 18th, 1997 is Monday by following the sequential order of Figure 2 (right). (right) A typical calendar of August, 1997. Classes by modulo 7 is given below the calendar. Compare it with finger sections of the right.

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1. Go to the finger section of the desired month in Figure 1.

2. Get the class by modulo 7 of the desired date. The class

   is the number of steps you use in step 3.

3. Starting from the section of the desired month, go steps according to the

   number you obtained in step 2.

4. According to the mapping between sections and days of the week as in Figure 2,

   reply the day of the section you reach in step 3, this is the answer.
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4 The Computational Account of "Finger Counting"

4.1 The Structural Accounts

We need two pieces of information:*the classification of a date to one of the classes by modulo 7 and the mapping between days and its classifiers* to obtain a day from a date as discussed in Section 2.3. In this section, we see how these information is represented and computed in "finger counting" in comparison with the case of a calendar.

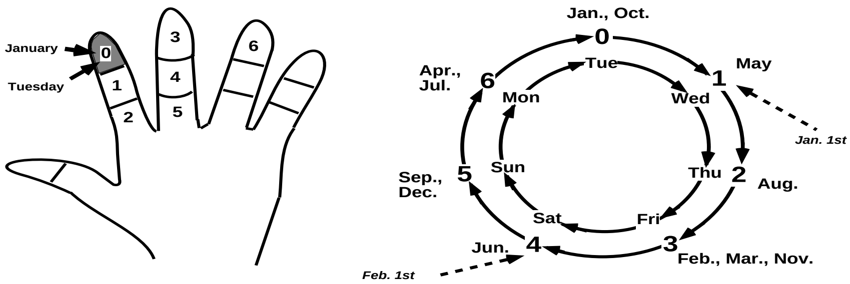

Figure 4: Computational Structure of "Finger Counting": (left) the mapping of dates of January to classes by modulo 7 onto sections. (right) The outer circle represents classes by modulo 7 for days in January. Each month of 12 months written outside of the circle stands for the correspondence of the beginning of each month with classes by modulo 7 of January. More precisely, 1st of each month corresponds with the next class of each month. The inner circle of days corresponds with classes by modulo 7 of January in the outer circle. This correspondence is of 1997. Each year, the correspondence between classes and days changes.

4.1.1 The Mapping of Months with Finger Sections

In "finger counting", the mapping of the desired month and the desired date with finger sections computationally classifies any date of any month into one of classes by modulo 7 of January. We explain how this classification is realized.

To understand this mapping, first note that January 1st, 1997 is Wednesday. This implies that the class '0' by modulo 7 of January is Tuesday and is allocated with the tip sector of the first finger as in Figure 4 (left). Other classes by modulo 7 and also other days of the week are aligned according to the same sequential order in Figure 2. Any date of January is classified as one of classes by modulo 7, that is, one of finger sections, so that the day of any date in January can be determined by using this alignment between classes by modulo 7 and finger sections.

What about February 1st? It is the next date after January 31th, *which is the class '3'*. In other words, February 1st is the class '4' *when counted from January 1st.* Then allocate February 1st with the class '4' *of January.* In other words, put the class '0' of February on the class '3' of January (Figure 1 (left) & in Figure 4). Then we can establish the alignment between classes by modulo 7 for any date of February and classes by modulo 7 of January, which allows us to determine the date of any date in February

, using the alignment between the days of the week and classes by modulo 7 of January. In a similar manner, we can consecutively make the alignment between classes of January and classes of any other month so that we can obtain a day from a date of any month. These procedures and knowledge are embedded in the mapping in Figure 1 (left).

4.1.2 The Relationship between Mappings of Months and Days

See Figure 4 (right). The outer circle represents the relationship of classes by modulo 7 of January with other 11 months, more precisely, with the class '0' by modulo 7 of each month. This relationship is invariant over years. In "finger counting", this relationship is embedded in the mapping between months and finger sections in Figure 1. Therefore, once learned, this mapping is kept for years.

The alignment between finger sections and the days of the week changes every year. Hence people have to reestablish the new alignment between sections and days every year. This means that one needs to rotate the inner circle in Figure 4 (right) accordingly. Since the sequential order of the days of the week, as in Figure 2, is so simple, it is quite easy to reconstruct the alignment once you know the mapping between any specific day and one of finger sections.

It is interesting to compare the treatment of dates and days in the "finger counting" procedure with that of a regular "calendar". In a calendar, the disposition of days in a month is invariant over months and over years in a calendar, meaning the convention that Sunday is always the most left column, which helps humans visually search for the desired date and visually map the date with the day of the week. In other words, the inner circle in Figure 4 is invariant in calendar. However, this forces people to *have an external representation for each month of each year.*

4.2 The Procedural Accounts

We summarized the general procedure in Section 3.2.2. Here, we explain a few aspects of this procedure that help humans use "finger counting", based on our preliminary observational studies. *Note that, though we provided the structural account of "finger counting" in advance, the computation of "finger counting" is learned*[9] *as the procedural steps without the explicit explanation of its computation.* There are *no* tags of months and days on finger sections.

The irregular mapping between a day and a date of a month of a year is realized by the superimposition of different computational structures (the date and day systems) onto an external representation: finger sections, in coordination with an internal manipulation: the calculation of an abstract class by modulo 7 for the desired date. Procedurally, this superimposition is implemented as two sequences of actions: (1) the finger tapping to get the section of the desired month and (2) the alignment between days and sections.

When people start to learn "finger counting", they first practice the sequential finger tapping. Learning the irregular mapping between days and dates is transformed into learning the sequential finger tapping, or sequential motor learning[1].[10]

To go to the desired month in step 1 of the general procedure, usually, people don't go directly to the month, in other words, they don't usually remember the mapping between months and sections "explicitly". They 'recall' the finger section of the desired month as they tap each section by their left thumb in the sequential order of months as in Figure 1. If they are interrupted, most of them start the sequence from January again. This implies that in memory, this mapping is represented by the finger tapping as a whole. *Recall of this irregular mapping "from memory" is transformed into the sequential finger tapping.*

While they calculate the class of the desired date (step 2) "in the head", their left thumb remains on the finger section of the desired month after step 1, and, then, they move their thumb a number of steps equal to the class. *Since the thumb is already at the class '0' of the desired month, it is quite easy to get this step* (step 3). *The thumb works as a marker[3] to "the external memory" while another computation is done "in the head".* When people are interrupted during counting steps, people usually begin to count again

[9] In Japan, it is also true. People learn not the structural account but only the procedure.

[10] The sequential order of months is not as regular as that of days. However, interestingly, it still has some regularities. It goes almost in a horizontal order except from November to December, descending vertically down rows.

from where they left their thumb. This implies that the the sequential tapping by the thumb is coordinated with counting the number of steps internally.

In step 4, people need to relate the mapping of the section they reach in step 3 with a day. People usually remember the mapping of the tip section of the first finger with the day of the week and align other finger sections with other days of the week. Since the sequential order of this alignment is quiten simple, this construction of the alignment each time is quite easily done. People remember this alignment as the procedure.[11]

5 Conclusion

The manipulation of external and internal representations are interwoven in cognitive processes. Whereas a calendar, as an external representation, is elaborated via the visual system, “finger counting” is an elaborated external representation coordinated with actions. A calendar takes the day system as invariant in order to facilitate visual search. In contrast, “finger counting” takes the date system as invariant. Procedurally, the computation of “finger counting” is implemented as sequences of actions. The “finger counting” procedure is supported by the human ability to superimpose different computational structures (i.e. months, dates, and days) onto external representations. “Finger counting” resolves the irregularity between the day and date systems by transforming it into sequential finger tapping. “Finger counting” illustrates how cognitive processes are internally and externally distributed and interwoven with each other.

Acknowledgments

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[11]Even if we forget the mapping between the tip sector of the first finger and the day of the week, we can simply reconstruct a whole alignment by taking account of the mapping of the day and date of “today”. The reconstruction of this alignment is very robust.

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